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# Modification of Einstein's Gravitational Field Equation and Cracking of Foundational Difficulties in Physics 

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Research Article

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#### Abstract

In the framework of gravitational theory of general relativity, this article has systematically and radically solved the problem of galaxy formation and some significant cosmological puzzles. A flaw with Einstein's equation of gravitational field is firstly corrected and the foundations of general relativity are perfected and developed, and space-time is proved to be infinite, expansion and contraction of universe are proved to be in circles, the singular point of big bang is naturally eliminated, celestial bodies and galaxies are proved growing up with cosmic expansion, for example Earth's mass and radius at present increase by 1.2 trillion tons and 0.45 mm , respectively in a year, in response to which geostationary satellites rise by 2.7 mm .


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## 1 Introduction

Though general relativity obtains considerable success, some significant problems such as the
problem of singular point, the problem of horizon, the problem of distribution and existence of dark matter and dark energy, the problem of the formation of celestial bodies and galaxies, the mystery of solar neutrino, as well as the problem of asymmetry of particle and antiparticle, always are not solved naturally and satisfactorily. These problems long remain implies strongly that the
fundamentals of general relativity have flaw and needs further perfection. For the purpose, this paper begins with determining the vacuum solution of Einstein's field equation in the background coordinate system, then by correcting rationally Einstein's field equation from an all new perspective these problems get removed.

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## 2 The static metric of spherical symmetry in background coordinate system

In this paper light's speed $c=1$. According to general relativity, for the static and spherically symmetric case, in the standard coordinate system (Weinberg, 1972; Peng and Xieng, 1998), the correct form of line element outside gravitational source is given by

$$
\begin{equation*}
d s^{2} \equiv-d \tau^{2}=-\left(1-\frac{2 G M}{l}\right) d t^{2}+\left(1-\frac{2 G M}{l}\right)^{-1} d l^{2}+l^{2}\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right) \tag{1}
\end{equation*}
$$

Here $\tau$ is proper time, M is the total mass of gravitational source; $l$ is called standard radial coordinate, doesn't have clear physical significance and only in the far field is approximately viewed as true radius. In order to describe clearly motion of particle and enable general relativity to link up with other theories of physics and to compare results one another, it is necessary to transform (1) into the form expressed in background coordinates. Hence we take $l=l(r)$. Here $r$ is defined as background coordinate (ZHou, 1982; Zhou, 1983; Fock, 1964) and refers to true radius which are said and used usually. $t, \theta, \varphi$ are standard coordinates and can also be viewed as background coordinates, which represent true time and angle. In the following we try to determine $l=l(r)$ by the introduction of an additional transformation equation, and such operation is allowed is because metric tensor satisfies Bianchi identity and if a metric is a solution of field equation in one coordinate system it is also a solution under arbitrary coordinate transformation.

According to general relativity the dynamical equation of particle outside source is geodesic

$$
\begin{equation*}
\frac{d^{2} x^{\mu}}{d t^{2}}+\Gamma_{\nu \lambda}^{\mu} \frac{d x^{\nu}}{d t} \cdot \frac{d x^{\lambda}}{d t}-\Gamma_{v \lambda}^{0} \frac{d x^{\nu}}{d t} \cdot \frac{d x^{\lambda}}{d t} \cdot \frac{d x^{\mu}}{d t}=0 \tag{2}
\end{equation*}
$$

where $x^{0}=t$, and indexes $\lambda, v, \mu, \sigma, \alpha, \beta=0,1,2,3$. Equation (2) can be proved as follows:
Using the usual form's geodesic $\frac{d^{2} x^{\mu}}{d s^{2}}+\Gamma_{\alpha \beta}^{\mu} \frac{d x^{\alpha}}{d s} \frac{d x^{\beta}}{d s}=0$, we have

$$
\frac{d^{2} x^{\mu}}{d s^{2}}=\frac{d t}{d s} \frac{d}{d t}\left(\frac{d t}{d s} \frac{d x^{\mu}}{d t}\right)=\left(\frac{d t}{d s}\right)^{2} \frac{d^{2} x^{\mu}}{d t^{2}}+\frac{d^{2} t}{d s^{2}} \frac{d x^{\mu}}{d t}=
$$

$\left(\frac{d t}{d s}\right)^{2}\left(\frac{d^{2} x^{\mu}}{d t^{2}}-\Gamma_{\alpha \beta}^{0} \frac{d x^{\alpha}}{d t} \frac{d x^{\beta}}{d t} \frac{d x^{\mu}}{d t}\right)$,
on the other hand, $\Gamma_{\alpha \beta}^{\mu} \frac{d x^{\alpha}}{d s} \frac{d x^{\beta}}{d s}=\left(\frac{d t}{d s}\right)^{2} \Gamma_{\alpha \beta}^{\mu} \frac{d x^{\alpha}}{d t} \frac{d x^{\beta}}{d t}$, and adding up yields immediately equation (2)

When a particle of mass $m$ is moving along radius in the static gravitational field of spherical symmetry, giving consideration to the speed, in the background coordinate system, in the far field (weak field) the radial component of Eq. (2) should reduce to the following relativistic equation (3) rather than others

$$
\begin{equation*}
\frac{d}{d t}\left[\left(\frac{d r}{d t}\right) m\right]=-\frac{m G M}{r^{2}} \tag{3}
\end{equation*}
$$

where $m$ refers to relativistic dynamic mass, namely $m=\frac{m_{0}}{\sqrt{1-v^{2}}}$. Why the radial component should reduce to (3) is that (3) stands for the equality of gravitational mass and inertial mass and also stands for the speed of light is the limit one. In order to enable it to reduce to (3) we may introduce a transformation equation as follows

$$
\begin{equation*}
\frac{d l}{d r}=\sqrt{1-\frac{2 G M}{l}} \exp \left(-\frac{G M}{r}\right) \tag{4}
\end{equation*}
$$

The correctness of Eq. (4) will be seen later, it determines a coordinate transformation of $l \rightarrow r$. By means of separating variables, the solution of Eq. (4) is easily given by

$$
\sqrt{l(l-2 G M)}+2 G M I n(\sqrt{l}+\sqrt{l-2 G M})=C_{1}+r-G M I n r-\frac{1}{2 r} G^{2} M^{2}+\frac{1}{12 r^{2}} G^{3} M^{3}+\cdots(
$$

Here constant $C_{1}$ is determined by the continuity of function $l=l(r)$ on boundary of source, and the back equation (23) can give out the boundary value $l\left(r_{e}\right), r_{e}$ denotes source's radius (celestial body radius). Note that $\Theta$ makes sure $l \approx r$ for $r \rightarrow \infty$, prove as follows
Form equation (4) we see $l \rightarrow \infty$ for $r \rightarrow \infty$, and considering of $\lim _{x \rightarrow \infty} \frac{\operatorname{In} x}{x}=0$, it holds that for $l \rightarrow \infty$ the left-hand side of $\Theta$ reads

$$
l\left(\sqrt{1-\frac{2 G M}{l}}-\frac{G M}{l} \operatorname{In} l-\frac{2 G M}{l} \operatorname{In}\left(1+\sqrt{1-\frac{2 G M}{l}}\right)\right) \approx l
$$

and for $r \rightarrow \infty$, the right-hand side of $\Theta$ is $r\left(\frac{C_{1}}{r}+1-\frac{G M}{r} \operatorname{Inr}-\frac{G^{2} M^{2}}{2 r^{2}}+\frac{G^{3} M^{3}}{12 r^{3}}+\cdots\right) \approx r$
Under transformation of Eq. (4), (1) becomes the following (5) which is an exact external solution expressed in background coordinates $r, t, \theta, \varphi$.

$$
\begin{equation*}
d s^{2}=-\left(1-\frac{2 G M}{l}\right) d t^{2}+\exp \left(-\frac{2 G M}{r}\right) d r^{2}+l^{2}\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right) \tag{5}
\end{equation*}
$$

Note that now $l=l(r)$ is already a specific function of $r$, which is determined by $\Theta$.
In the far field, the line element (5) provides:
$g_{00}=-1+\frac{2 G M}{l(r)} \approx-1+\frac{2 G M}{r}, \quad g_{11}=\exp \left(-\frac{2 G M}{r}\right) \approx 1-\frac{2 G M}{r}, \quad g_{22}=l^{2}(r) \approx r^{2}$,
$g_{33}=l^{2}(r) \sin ^{2} \theta \approx r^{2} \sin ^{2} \theta, \Gamma_{00}^{1} \approx \frac{G M}{r^{2}}, \Gamma_{11}^{1} \approx \frac{G M}{r^{2}}, \Gamma_{01}^{0} \approx \frac{G M}{r^{2}}, \Gamma_{01}^{1} \approx 0, \Gamma_{00}^{0}=0$,
and introducing them into (2) and putting $\mu=1, d \theta=d \varphi=0, v=\frac{d r}{d t}$,
we obtain

$$
\begin{equation*}
\frac{d^{2} r}{d t^{2}}+\left(1-v^{2}\right) \frac{G M}{r^{2}}=0 \tag{6}
\end{equation*}
$$

which is equivalent to Eq. (3). Proof: assume $d \theta=d \varphi=0, \quad m=\frac{m_{0}}{\sqrt{1-v^{2}}}$, from equation (3) we have

$$
\begin{aligned}
0 & =\frac{d}{d t}\left[\left(\frac{d r}{d t}\right) m\right]+\frac{m G M}{r^{2}}=m_{0}\left[v \frac{d\left(1-v^{2}\right)^{-1 / 2}}{d t}+\left(1-v^{2}\right)^{-1 / 2} \frac{d v}{d t}\right]+\frac{m G M}{r^{2}} \\
& =\left(1-v^{2}\right)^{-3 / 2} \frac{d^{2} r}{d t^{2}} m_{0}+m \frac{G M}{r^{2}}=m\left[\left(1-v^{2}\right)^{-1} \frac{d^{2} r}{d t^{2}}+\frac{G M}{r^{2}}\right],
\end{aligned}
$$

which immediately yields (6).
Consequently, we conclude that (5) is the appropriate line element which satisfies the requirements completely.

As a serious emphasis, we must point out that using directly $l=r$ in (1) gives another exact solution, namely the following (7)

$$
\begin{equation*}
d s^{2}=-\left(1-\frac{2 G M}{r}\right) d t^{2}+\left(1-\frac{2 G M}{r}\right)^{-1} d r^{2}+r^{2}\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right) \tag{7}
\end{equation*}
$$

However, in accordance with (7) the corresponding geodesic can't reduce to (3) in weak field, instead it reduces to

$$
\begin{equation*}
\frac{d^{2} r}{d t^{2}}+\left(1-3 v^{2}\right) \frac{G M}{r^{2}}=0 \tag{8}
\end{equation*}
$$

## Proof:

(7) provides $g_{00}=-1+\frac{2 G M}{r}, g_{11}=\left(1-\frac{2 G M}{r}\right)^{-1}, g_{22}=r^{2}, g_{33}=r^{2} \sin ^{2} \theta, g_{\mu v}=0(\mu \neq v)$

$$
\Gamma_{11}^{1}=\frac{1}{2} g^{1 \rho}\left(\frac{\partial g_{\rho 1}}{\partial x^{1}}+\frac{\partial g_{\rho 1}}{\partial x^{1}}-\frac{\partial g_{11}}{\partial x^{\rho}}\right)=-\frac{G M}{(1-2 G M / r) r^{2}}, \Gamma_{01}^{0}=\frac{G M}{(1-2 G M / r) r^{2}},
$$

$\Gamma_{00}^{1}=\frac{(1-2 G M / r) G M}{r^{2}} \quad \Gamma_{01}^{1}=0, \quad$ substituting them into (2) and taking $\mu=1$ and $d \varphi=d \theta=0$ yield immediately

$$
\frac{d^{2} r}{d t^{2}}=-\Gamma_{00}^{1}-\Gamma_{11}^{1} v^{2}+2 v^{2} \Gamma_{01}^{0}=-\left(1-\frac{2 G M}{r}\right) \frac{G M}{r^{2}}+\frac{3 G M}{(1-2 G M / r) r^{2}} v^{2}, \quad \text { and } \quad \text { for }
$$

$\frac{2 G M}{r} \ll 1$, this equation obviously reduces to Eq. (8), which isn't Eq. (3). It is easily found that Eq. (8) not only goes against the elementary principle of equality of gravitational mass and inertial
mass but also leads to incorrect conclusion that gravitational field becomes repulsive one for a particle whose speed exceeds 0.58 c . Hence Eq. (8) must be wrong, and implies (7) can’t describe high speed and has shortcoming as compared with (5).
Note that the angle of orbital precession of Mercury described by (5) is still the same as that described by line element (7) (Peng and Xieng, 1998), it doesn't change under the transformation of radial coordinates. In a word, (5) is the correct line element expressed in background coordinate system
And again, though general relativity doesn't exclude to use other coordinates, we must use background coordinates when we take geodesic to compare with Newtonian gravitational law because the meaning of every term in the geodesic is not clear in other coordinates and the comparison cannot realize.

## 3 Modification of Einstein's gravitational field equation

It is seen from the above discussions that in the case of weak field approximation, $g_{00}=-1+\frac{2 G M}{r}$ and $g_{11}=1-\frac{2 G M}{r}$ instead of the previous $g_{11}=1+\frac{2 G M}{r}$, which are just the requirement of that (6) can hold and hint us to alter the coupling constant $\gamma$ in gravitational field equation $R_{\mu \nu}=\gamma\left(T_{\mu \nu}-\frac{1}{2} T g_{\mu \nu}\right)$. Note that the coupling constant $\gamma$ relates to the form of weak field approximation metrics $g_{\mu \nu}$ and is determined in the course of solving weak field approximation metrics, and the change of the metrics means that the coupling constant $\gamma$ need change too. And now we set out to reconfirm the coefficient $\gamma$ by solving weak field approximation metrics $g_{\mu \nu}$.

Here $T_{\mu \nu} \equiv(\rho+p) U_{\mu} U_{\nu}+p g_{\mu \nu}$, and $U^{\mu} \equiv \frac{d x^{\mu}}{d \tau}, U_{\mu} \equiv g_{\mu \nu} U^{\nu}$.
And from $d s^{2}=-d \tau^{2}=g_{\mu \nu} d x^{\mu} d x^{\nu}$, we have $U_{\mu} U^{\mu}=-1$, then it follows that

$$
T=g^{\mu v}(\rho+p) U_{\mu} U_{v}+p g^{\mu v} g_{\mu v}=(\rho+p) U_{\mu} U^{\mu}+4 p=3 p-\rho
$$

Similar to previous calculation to appear in standard textbooks, the following discussions are still carried out in right-angled coordinate system. For weak field we have $g_{\mu \nu}=h_{\mu \nu}+\eta_{\mu \nu}$ and $\left|h_{\mu v}\right| \ll 1$. Here Minkowskian metrics $\eta_{00}=-1, \eta_{11}=\eta_{22}=\eta_{33}=1, \eta_{\mu \nu}=0(\mu \neq v)$.

Omitting the terms of less than $O\left(h^{2}\right)$ we have (Weinberg, 1972)
$\Gamma_{\alpha \beta}^{\mu}=\frac{1}{2} \eta^{\mu \rho}\left(\frac{\partial g_{\rho \alpha}}{\partial x^{\beta}}+\frac{\partial g_{\rho \beta}}{\partial x^{\alpha}}-\frac{\partial g_{\alpha \beta}}{\partial x^{\rho}}\right), h_{\beta}^{\mu}=\eta^{\mu \rho} h_{\rho \beta}$, and $h=h_{\mu}^{\mu}=\eta^{\mu \rho} h_{\mu \rho}$.
Correspondingly, Rich tensor

$$
R_{\mu \nu}=\Gamma_{\mu \sigma, v}^{\sigma}-\Gamma_{\mu \nu}^{\sigma}, \sigma=\frac{1}{2} \eta^{\sigma \lambda} h_{\mu v, \lambda, \sigma}+\frac{1}{2}\left(h,{ }_{\mu},_{v}-h_{\mu, \lambda,{ }_{v}}^{\lambda}-h_{v, \sigma}^{\sigma},_{\mu}\right) .
$$

Note that $A,{ }_{\mu} \equiv \frac{\partial A}{\partial x^{\mu}}$.
May as well use harmonic condition

$$
\begin{equation*}
h_{\mu}^{\sigma},_{\sigma}=\frac{1}{2} h,_{\mu} . \tag{9}
\end{equation*}
$$

Differentiating Eq. (9) with respect to $x^{\nu}$ yields $h_{\mu}^{\sigma}, \sigma{ }_{\sigma}=\frac{1}{2} h,{ }_{\mu},_{v}$. Similarly, we have $h_{v}^{\sigma},_{\sigma},{ }_{\mu}=\frac{1}{2} h,{ }_{v},{ }_{\mu}$. Using $h,{ }_{v},{ }_{\mu}=h,{ }_{\mu},{ }_{v}$ and adding up the above two equations yield $h,{ }_{\mu},{ }_{v}-h_{\mu}^{\lambda},{ }_{\lambda},{ }_{v}-h_{v}^{\sigma}, \sigma,{ }_{\mu}=0$. Hence, we obtain

$$
\nabla^{2} h_{\mu \nu}-\frac{\partial^{2} h_{\mu \nu}}{\partial t^{2}}=2 \gamma\left(T_{\mu \nu}-\frac{1}{2} T \eta_{\mu \nu}\right)=2 \gamma\left[(\rho+p) U_{\mu} U_{\nu}+\frac{\rho-p}{2} \eta_{\mu \nu}\right],
$$

which have retarded solutions $h_{\lambda \lambda}=-\frac{\gamma}{4 \pi} \int \frac{2(\rho+p) U_{\lambda}^{2}+(\rho-p) \eta_{\lambda \lambda}}{\xi} d x^{\prime} d y^{\prime} d z^{\prime}$.
Here $\xi=\sqrt{\left(x-x^{\prime}\right)^{2}+\left(y-y^{\prime}\right)^{2}+\left(z-z^{\prime}\right)^{2}}, \quad i, j, k=1,2,3$, the terms in the integral sign take the values of $t^{\prime}=t-\xi$. Note that the above retarded solutions can be used in arbitrary cases of motion of source. Hence, in order to get the external metrics $g_{00}=-1+\frac{2 G M}{r}$ and $g_{j j}=1-\frac{2 G M}{r}$ in the case of static spherical symmetry $\left(U_{0}=\eta_{0, \mu} U^{\mu}=-1, U_{j}=0\right)$, it must be required that the constant coefficient $\gamma=4 \pi G$ and simultaneously pressure $p$ satisfies

$$
\begin{gather*}
\int \frac{p}{\xi} d x^{\prime} d y^{\prime} d z^{\prime}=-\int \frac{\rho}{\xi} d x^{\prime} d y^{\prime} d z^{\prime}=-\frac{M}{r} \text { for } r=\sqrt{x^{2}+y^{2}+z^{2}} \geq r_{e} \text {, which means } \\
\int p d x d y d z=-\int \rho d x d y d z=-M . \tag{10}
\end{gather*}
$$

In view of (3) it must hold that $h_{0 j}=0$ in the static case. Next we solve for the other three $h_{i j}$. Inserting $h_{\mu}^{\sigma}=\eta^{\sigma \lambda} h_{\lambda \mu}$ and $h=\eta^{\nu \lambda} h_{\nu \lambda}=-h_{00}+3 h_{11}$ into (9), and noticing $h_{11}=h_{22}=h_{33}$, $h_{j}^{i}=h_{i j}=h_{j i}=h_{i}^{j}, h_{0}^{i}=h_{0 i}=0, h_{i}^{0}=-h_{0 i}=0$, we obtain three equations as follows

$$
\begin{aligned}
& h_{13,1}+h_{23}, 2=\frac{1}{2}\left(h_{11}-h_{00}\right),_{3} \\
& h_{12},{ }_{1}+h_{23}, 3=\frac{1}{2}\left(h_{11}-h_{00}\right),_{2} \\
& \left.h_{12,2}+h_{13,3}=\frac{1}{2}\left(h_{11}-h_{00}\right)\right)_{1}
\end{aligned}
$$

After a certain calculation we arrive in

$$
\left.h_{i j},_{i},,_{j}=\frac{1}{4}\left[\left(h_{11}-h_{00}\right)\right)_{i},_{i}+\left(h_{11}-h_{00}\right),_{j},,_{j}-\left(h_{11}-h_{00}\right),_{k},{ }_{k}\right]
$$

Here $i \neq j, i \neq k \quad k \neq j$, and $i, j, k=1,2,3$. With the condition $h_{i j} \rightarrow 0$ for $r \rightarrow \infty, h_{i j}$ are solved by

$$
h_{j i}=\frac{1}{4} \int_{\infty}^{x^{j}} \int_{\infty}^{x^{i}}\left[\left(\frac{\partial^{2}}{\partial\left(x^{i}\right)^{2}}+\frac{\partial^{2}}{\partial\left(x^{j}\right)^{2}}-\frac{\partial^{2}}{\partial\left(x^{k}\right)^{2}}\right)\left(h_{11}-h_{00}\right)\right] d x^{j} d x^{i}
$$

Note that $x^{1}=x, x^{2}=y, x^{3}=z$. On the other hand, for the weak field case Bianchi identity can give the ordinary conservation law $T_{v, \mu}^{\mu}=0$.

## Proof:

$R_{v ; \mu}^{\mu}=R_{v, \mu}^{\mu}+\Gamma_{\lambda \mu}^{\mu} R_{v}^{\lambda}-\Gamma_{\lambda v}^{\mu} R_{\mu}^{\lambda}=R_{v, \mu}^{\mu}+o\left(h^{2}\right)=R_{v, \mu}^{\mu}$,
then $0=\left(R_{v}^{\mu}-\frac{1}{2} R \delta_{v}^{\mu}\right)_{; \mu}=R_{v ; \mu}^{\mu}-\frac{1}{2} R_{; \nu}=R_{v, \mu}^{\mu}-\frac{1}{2} R_{, v}$,
moreover field equation gives
$R=-\gamma T$ and $R_{v, \mu}^{\mu}=\gamma\left(T_{v}^{\mu}-\frac{1}{2} T \delta_{v}^{\mu}\right)_{, \mu}=\gamma\left(T_{v, \mu}^{\mu}-\frac{1}{2} T_{, v}\right)=\gamma T_{v, \mu}^{\mu}+\frac{1}{2} R_{, v}$,
hence $T_{v, \mu}^{\mu}=0$.
For the static case, using $T_{v, \mu}^{\mu}=\left[(\rho+p) U_{\nu} U^{\mu}\right]_{, \mu}+\left(p \delta_{v}^{\mu}\right)_{, \mu}=0$ yields $\frac{\partial p}{\partial x^{\nu}}=0$, considering of $\nabla^{2}\left(h_{00}-h_{\mathrm{jj}}\right)=16 \pi G p$, it is verified that

$$
\nabla^{2} h_{j i}=\frac{1}{4} \int_{\infty}^{x^{j}} \int_{\infty}^{x^{i}}\left[\left(\frac{\partial^{2}}{\partial\left(x^{i}\right)^{2}}+\frac{\partial^{2}}{\partial\left(x^{j}\right)^{2}}-\frac{\partial^{2}}{\partial\left(x^{k}\right)^{2}}\right) \nabla^{2}\left(h_{11}-h_{00}\right)\right] d x^{j} d x^{i}=0
$$

That is to say, $h_{v \mu}$ worked out here is indeed reasonable approximate solution of field equation with $\gamma=4 \pi G$.

And again, as a special case of spherical symmetry, namely $\frac{\partial \rho}{\partial x^{v}}=0$, since $\frac{\partial p}{\partial x^{v}}=0$ we infer from (10) a very useful result

$$
p=-\rho
$$

which can be regarded as the form of pressure in weak field in the case of that $\rho$ is homogeneous. It is obviously too subjective to take pressure for zero in advance, in fact, by serious calculation we see that the pressure is negative where matter exists and the place where matter exists turns out to be so-called pseudo-vacuum (Gondolo and Fresse, 2003; Guth, 1981). This is a new important result which isn't in agreement with traditional opinion.
To sum up, we can conclude that in any coordinate system gravitational field equation is revised as

$$
\begin{equation*}
R_{\mu \nu}-\frac{1}{2} R g_{\mu \nu}=4 \pi G T_{\mu \nu}, \tag{11}
\end{equation*}
$$

where 4 replaces previous -8 , obviously Eq. (11) preserves general covariance. Of course, (1) and (5) satisfy Eq. (11) because both $p$ and $\rho$ vanish outside source, Eq. (11) becomes $R_{\mu \nu}=0$.

## 4 Applications and tests of Eq. (11) in cosmology

With $l$ as standard radial coordinate, in the co-moving coordinates Friedmann-Robertson-Walker metric is given by (Weinberg, 1972; Sawangwit and Shanks, 2005).

$$
d s^{2}=-d t^{2}+a^{2}(t)\left[\frac{1}{1-k l^{2}} d l^{2}+l^{2} d \theta^{2}+l^{2} \sin ^{2} \theta d \varphi^{2}\right]
$$

$a(t)$ is expansion factor. $g_{00}=-1, g_{11}=\frac{a^{2}(t)}{1-k l^{2}}, g_{22}=a^{2}(t) l^{2}, \quad g_{33}=a^{2}(t) l^{2} \sin ^{2} \theta$, $g_{\mu \nu}=0(\mu \neq v)$, and substituting they into (11) yields

$$
\begin{equation*}
\left(\frac{d a(t)}{d t}\right)^{2}+k=-\frac{4 \pi G}{3} \rho a^{2}(t) \tag{12}
\end{equation*}
$$

Consequently $k$ must be negative, cosmos is so far proved infinite or open. And again, in virtue of $T_{; \beta}^{\alpha \beta}=\left(n U^{v}\right)_{; v}=\left(U_{\mu} U^{\mu}\right)_{; \beta}=2 U_{\mu}\left(U^{\mu}\right)_{; \beta}=2 U^{\mu}\left(U_{\mu}\right)_{; \beta}=0, \quad$ it follows that $d\left(\rho a^{3}\right)+p d a^{3}=0$ and

$$
\begin{equation*}
p d\left(\frac{1}{n}\right)+d\left(\frac{\rho}{n}\right)=0 \tag{13}
\end{equation*}
$$

Here n represents the density of particle (galaxy) number. Since $\rho$ is assumed homogeneous, we may use the weak field condition $p=-\rho$, and substituting it into Eq. (13) yields $d \rho=0$, that is to say, $\dot{p}=-\dot{\rho}=0$ or

$$
\begin{equation*}
p=-\rho=\text { const }=-\rho_{0}, \tag{14}
\end{equation*}
$$

which is the most appropriate expression of energy conservation in infinite spacetime and indicates the singular point of big bang did not exist. In addition, (13) implies the mass of galaxy changing with cosmic expansion since $\rho / n$ stands for per particle mass. And further, the solution of Eq. (12), namely expanding factor, is given by

$$
\begin{equation*}
a(t)=A \sin \left(t \sqrt{\frac{4 \pi G \rho_{0}}{3}}\right) \tag{15}
\end{equation*}
$$

Here $A$ is a positive constant. So far cosmic expansion and contraction are proved to be in circles. Note that (15) shows that the expansion of the present universe is decelerating but not accelerating this fact agrees to the newest conclusions (Dominik J, 1993).
Now we compute the relation between distance and red-shift. May as well put $a\left(t_{0}\right)=1$, the light from a galaxy to us satisfies (Weinberg,1972) $1+z=\frac{1}{a(t)}$ and $d z=-\frac{d a}{a^{2}(t)}$.
Here z denotes red-shift. And writing $\frac{4 \pi G \rho_{0}}{3 H_{0}^{2}}=q_{0}, H\left(t_{0}\right)=H_{0}$, we infer from Eq. (12)

$$
H \equiv a^{-1} \frac{d a}{d t}=H_{0} \sqrt{\left(1+q_{0}\right)(1+z)^{2}-q_{0}}, \quad k=-H_{0}^{2}\left(1+q_{0}\right) .
$$

Note that the subscript " 0 " refers to the present-day values. For the propagation of light line $d s^{2}=0$, then $\frac{d t}{a(t)}=-\frac{d z}{H}=-\frac{d l}{\sqrt{\left(1-k l^{2}\right)}}, \int_{0}^{Z} \frac{d z}{H}=\int_{0}^{l_{a}} \frac{d l}{\sqrt{1-k l^{2}}} . l_{a}$ denotes the galaxy's invariant coordinate. In view of luminosity-distance $d_{L}=(1+z) \int_{0}^{l_{a}} \frac{d l}{\sqrt{1-k l^{2}}}$, we work out a new relation between distance and re-shift

$$
\begin{equation*}
H_{0} d_{L}=\frac{z+1}{\sqrt{q_{0}+1}} \operatorname{In} \frac{(z+1) \sqrt{q_{0}+1}+\sqrt{\left(q_{0}+1\right)(z+1)^{2}-q_{0}}}{1+\sqrt{q_{0}+1}} \tag{16}
\end{equation*}
$$

As $Z \rightarrow 0$, expanding the right hand side of (16) into power series with respect of $Z$, (16) becomes

$$
H_{0} d_{L}=z+\frac{1-q_{0}}{2} z^{2}+\frac{3 q_{0}^{2}-2 q_{0}-1}{6} z^{3}+\cdots,
$$

which is the same result as that obtained via pure kinematics. The curved line in figure 1 (Dai zi Gao, 2005) is the image of (16) with $q_{0}=0.14$ and $H_{0}=70 \mathrm{~km} \cdot \mathrm{~s}^{-1} \cdot M p c^{-1}$. The situation described by the curved line agrees well with the recent data of observations. Note that recent observations give $q_{0}=\frac{4 \pi G \rho}{3 H_{0}^{2}} \equiv \frac{\Omega_{0}}{2}=0.1 \pm 0.05$. (Linder, 2003; Hamuy, 2003; Alcaniz, 2004 )


Fig. 1. The Recent Hubble diagram
Note that the spots in the figure 1 represent galaxies. Distance-Modulus is equal to $5 \lg d_{L}+25$ , and the unit of $d_{L}$ is Mpc Next we calculate "our" cosmic age, namely the time from last $a(t)=0$ (at the moment, $t$ may as well take 0 ) to today. Writing $H\left(t_{0}\right)=H_{0}$, from $H=\frac{\dot{a}}{a}=2 \sqrt{\frac{\pi G \rho}{3}} \operatorname{ctg}\left(2 t \sqrt{\frac{\pi G \rho}{3}}\right)$, in the case that $q_{0}$ takes 0.14 "our" cosmic age is calculated as

$$
\begin{equation*}
t_{0}=\frac{\operatorname{tg}^{-1} \sqrt{q_{0}}}{H_{0} \sqrt{q_{0}}}=1.37 \times 10^{10} a \tag{17}
\end{equation*}
$$

which agrees with observations. Besides, we can also compute how a galaxy's mass changes with time. Writing a galaxy' mass $m(t)$, taking account of $\rho=$ const $=N m(t) / a^{3}(t)$, where N is equivalent to a proportional coefficient, immediately it is concluded that

$$
\begin{equation*}
\frac{m\left(t_{1}\right)}{a^{3}\left(t_{1}\right)}=\frac{m\left(t_{2}\right)}{a^{3}\left(t_{2}\right)}, \tag{18}
\end{equation*}
$$

which implies that galaxies can grow up without mergers and consists with recent observations. The formula (18) defines how a galaxy mass changes with evolution of universe.

And again, because any point can be thought the centre of universe's expansion, (18) can be looked as the rule of mass's change of any celestial body or galaxy. And applying (18) to the earth of today, we find that the increase of the earth's mass in a year is

$$
\begin{equation*}
\Delta m_{0}=\left[\frac{a^{3}\left(t_{0}+1\right)}{a^{3}\left(t_{0}\right)}-1\right] m\left(t_{0}\right) \approx 3 H_{0} m_{0}=12.46 \times 10^{14} \mathrm{~kg} \tag{19}
\end{equation*}
$$

And also deduce that the expanding speed of the radius of the earth is today $v_{0}=H_{0} r_{0}=0.45 \mathrm{~mm} / a$.

By the way, from $a\left(t_{0}\right)=A \sin \left(t_{0} \sqrt{\frac{4 \pi G \rho_{0}}{3}}\right)=1$ we can decide constant
$A=1 / \sin \left(t_{0} \sqrt{\frac{4 \pi G \rho_{0}}{3}}\right)$, and further we have the following relation of reshift Z and universe time $t$

$$
1+z=\frac{1}{a(t)}=\sin \left(t_{0} \sqrt{\frac{4 \pi G \rho_{0}}{3}}\right) / \sin \left(t \sqrt{\frac{4 \pi G \rho_{0}}{3}}\right)
$$

Here $t$ is the time at which photons was given out from the celestial body. The relation can be used to evaluate low limit of celestial body age.

## 5 Exact interior solution of Eq. (11) and mechanism of celestial body's expansion.

In the case of static spherical symmetry, inside a celestial body (gravitational source), with $l$ as standard radial coordinate the exact interior solution of Eq. (11) is given by .

$$
\begin{align*}
d s^{2}= & -\exp \left[\mathrm{C}_{2}+\int_{l}^{l_{l}} f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1} d l\right] d t^{2} \\
& +\left(1+\frac{G \omega(l)}{l}\right)^{-1} d l^{2}+l^{2}\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right) \tag{20}
\end{align*}
$$

in which $\omega(l) \equiv 4 \pi \int_{0}^{l} \rho(l) l^{2} d l, f(l) \equiv \frac{G}{l^{2}}\left[4 \pi l^{3} p(l)+\omega(l)\right]$, and $l_{e}=l\left(r_{e}\right)$. And constant $C_{2}=\operatorname{In}\left[1-\frac{2 G M}{l_{e}}\right]$, it makes sure $g_{00}$ is continual on the boundary of the celestial body. Note that as scalar $\rho=\rho(l)=\tilde{\rho}(r), \quad p=p(l)=\tilde{p}(r)$, and outside gravitational source both $p$ and $\rho$ vanish, namely $\rho(l)=\tilde{\rho}(r)=\tilde{p}(r)=p(l)=0$ for $r>r_{e}$.

In order to determine the interior form of (20) in background coordinates, Eq. (4) is naturally extended as inside source

$$
\begin{equation*}
\frac{d l}{d r}=\sqrt{1+\frac{G \omega(l)}{l}} \exp \left(-G \int \frac{\rho}{\xi} d x^{\prime} d y^{\prime} d z^{\prime}\right) . \tag{21}
\end{equation*}
$$

Obvious under the transformation of Eq. (21), line element (20) turns into
$d s^{2}=-\exp \left[\mathrm{C}_{2}+\int_{l}^{l_{e}} f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1} d l\right] d t^{2}+$

$$
\begin{equation*}
\exp \left(-2 G \int \frac{\rho}{\xi} d x^{\prime} d y^{\prime} d z^{\prime}\right) d r^{2}+l^{2}\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right) \tag{22}
\end{equation*}
$$

Here $l=l(r)$ is a specific function of $r$, which is determined by Eq. (21). Line element (22) is just the exact solution looked for and expressed in background coordinates $r, t, \theta, \varphi$. Note that the solution of Eq. (21) satisfy the initial condition $l(0)=0$. In fact, because there is no acceleration tendency for every direction at the centre gravitational source, $d g_{00} / d r$ must be zero, and from (22) we have

$$
0=\frac{d g_{00}}{d r}=\frac{d l}{d r} \frac{d g_{00}}{d l}=\frac{d l}{d r} f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1} \exp \left[\mathrm{C}_{2}+\int_{l}^{l_{e}} f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1} d l\right]
$$

which indicates $f(l)=0$ at the centre, and so that $l=l(0)=0$ at the centre.
And if $\rho=$ const $=\frac{3 M}{4 \pi r_{e}^{3}}$, then we have
$\int \frac{\rho}{\xi} d x^{\prime} d y^{\prime} d z^{\prime}=\frac{3 M}{2 r_{e}}-\frac{M}{2 r_{e}^{3}} r^{2}, \quad \omega(l)=4 \pi \int_{0}^{l} \rho(l) l^{2} d l=\frac{M}{r_{e}^{3}} l^{3}$, the solution of Eq. (21) is easily given by
$\sqrt{\frac{r_{e}^{3}}{G M}} \operatorname{In}\left(\sqrt{\frac{G M}{r_{e}^{3}}} l+\sqrt{1+\frac{G M}{r_{e}^{3}} l^{2}}\right)=\left[r+\frac{G M}{6 r_{e}^{3}} r^{3}+\frac{1}{40}\left(\frac{G M}{r_{e}^{3}}\right)^{2} r^{5}+\cdots\right] \exp \left(-\frac{3 G M}{2 r_{e}}\right)$
Though $\rho$, generally speaking, isn't constant, we may take its average value or piecewise integrate in practice for the convenience of calculation. For example, on the surface of Sun $r=r_{e}=6.96 \times 10^{8} \mathrm{~m}, M=1.99 \times 10^{30} \mathrm{~kg}$, using (23) namely taking average value of $\rho$ we can work out the surface's $l=l\left(r_{e}\right)=6.96 \times 10^{8} \mathrm{~m}-1720 \mathrm{~m}$, which is highly equal to Sun's radius. And likewise, we can work out $l=6371 \mathrm{~km}-0.00038 \mathrm{~km}$ on Earth's surface, and this almost equals Earth's radius ( 6371 km ).

So far, using the continuity of $l=l(r)$ not only we can determine the constant $C_{1}$ but also can calculate the deflected angle of light line on the surface of Sun. For photon's propagation outside Sun from (5) we have

$$
\begin{aligned}
0=d s^{2} & =-\left(1-\frac{2 G M}{l}\right) d t^{2}+\exp \left(-\frac{2 G M}{r}\right) d r^{2}+l^{2}\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right) \\
& =-\left(1-\frac{2 G M}{l}\right) d t^{2}+\left(1-\frac{2 G M}{l}\right)^{-1} d l^{2}+\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right) l^{2}
\end{aligned}
$$

Similar to former calculation, the deflected angle is given by $\alpha=\frac{4 M G}{l}=\frac{4 M G}{l\left(r_{e}\right)}=1.78^{\prime \prime}$, which is more consistent with observational result (1.89") compared with former theoretical value $\alpha=\frac{4 M G}{r}=\frac{4 M G}{r_{e}}=1.75^{\prime \prime}$.
On the other hand, the conserved law gives out

$$
\begin{equation*}
\frac{d p}{d l}=G(p+\rho)\left(2 \pi l^{3} p+\frac{\omega}{2}\right)\left(l^{2}+l G \omega(l)\right)^{-1} \tag{24}
\end{equation*}
$$

On the boundary the gravity acceleration should be continual, according to (2), using (4), (5), (21), (22) we have

$$
\left(\Gamma_{00}^{1}\right)_{r=r_{e}^{+}}=\left(\Gamma_{00}^{1}\right)_{r=r_{e}^{-}}, \text {that is, }\left(g^{11} \frac{d g_{00}}{d r}\right)_{r=r_{e}^{+}}=\left(g^{11} \frac{d g_{00}}{d r}\right)_{r=r_{e}^{-}},
$$

it follows that

$$
\left[\frac{d l}{d r} \frac{d}{d l}\left(1-\frac{2 G M}{l}\right)\right]_{r=r_{e}^{+}}=\left\{\frac{d l}{d r} \frac{d}{d l} \exp \left[\mathrm{C}_{2}+\int_{l}^{l_{e}} f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1} d l\right]\right\} r=r_{e}^{-}
$$

And after simplifying further, it becomes

$$
\begin{equation*}
\left[4 \pi l_{e}^{3} p+\omega\left(l_{e}\right)\right] \sqrt{l_{e}-2 G M}=-2 M \sqrt{l_{e}+G \omega\left(l_{e}\right)} \tag{25}
\end{equation*}
$$

which is the boundary condition $p$ must satisfy, and the condition determines $p$ negative within celestial body.

For general cases, inside source, gravitational field is still weak, which means $l=l(r) \approx r$, $\frac{2 G M}{r} \ll 1$, and from (25) the boundary pressure $p \approx-\frac{3 M}{4 \pi r_{e}^{3}}=-\bar{\rho}$, which is consistent with (10). Here $\bar{\rho}$ denotes the average of matter density

As an emphasis, we must point out that when (1) or (5) is applied to a mass point of the surface of the static source, it holds that $0 \geq d s^{2}=-\left(1-\frac{2 G M}{l}\right) d t^{2}$, which indicates that $1-\frac{2 G M}{l}$ of static source is nonnegative.

Next let us consider a small volume $V_{i}$ of mass $m_{i}$ inside source, $d V_{i}$ denotes $V_{i}$ 's change caused from the expansion of space-time, in view of Eq. (12) we have $d m_{i}=-p_{i} d V_{i}$, hence

$$
d \rho_{i}=d\left(\frac{m_{i}}{V_{i}}\right)=-\left(\rho_{i}+p_{i}\right) \frac{d V_{i}}{V_{i}}=-\left(\rho_{i}+p_{i}\right) \frac{d a^{3}(t)}{a^{3}(t)} \text {, which means that for arbitrary }
$$

point it holds that

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}=-\frac{\rho+p}{a^{3}(t)} \cdot \frac{d a^{3}(t)}{d t} \tag{26}
\end{equation*}
$$

(26) determines how matter density changes locally. It is seen from (26) that when celestial bodies expand with cosmic expansion its density may be unchanging in the case of $\rho+p=0$.
So far, we deduce that bursts of celestial bodies and formation of earthquakes originate from unceasing accumulation of inside matter and change of distribution; and it is the negative pressure that gets matter in celestial body continuously product.

## 6 Cracking of the problem of dark matter

The negative pressure as important gravitational source is invisible, and it is the negative pressure that appears as the form of dark matter and leads to the phenomenon of missing mass, or say that so-called dark matter is just the negative pressure, this fact is showed as follows.

Speaking generally, within a galaxy the metric field is weak field, and when a galaxy is treated as a celestial body of spherical symmetry, according to the discussion in section 3, within the galaxy ( $0 \leq r \leq r_{e}$ ) pressure $p=$ const $\neq 0$. And from (10) we infer $p=$ const $=-\frac{3 M}{4 \pi r_{e}^{3}}$, and further we have

$$
\begin{aligned}
h_{00}= & -G \int \frac{\rho+3 p}{\xi} d x^{\prime} d y^{\prime} d z^{\prime}= \\
& -4 \pi G\left(r^{-1} \int_{0}^{r} \rho r^{2} d r+\int_{0}^{r_{e}} \rho r d r-\int_{0}^{r} \rho r d r\right)-6 G \pi p r_{e}^{2}+2 G \pi p r^{2}
\end{aligned}
$$

According to (2) the gravity acceleration (or gravitational field strength) within the galaxy is given by

$$
g=-\Gamma_{00}^{1}=\frac{1}{2} \frac{d h_{00}}{d r}=2 \pi G p r+\frac{2 \pi G}{r^{2}} \int_{0}^{r} \rho r^{2} d r=2 \pi G p r+\frac{G m(r)}{2 r^{2}}
$$

where $m(r) \equiv 4 \pi \int_{0}^{r} \rho r^{2} d r$, and $g$ may be positive or negative since pressure is negative, and the negative $g$ indicates the direction of acceleration is towards centre. And according to (2) the corresponding round orbital speed $V_{T}$ satisfies

$$
\begin{equation*}
v_{T}^{2}=-g r=-2 \pi G p r^{2}-\frac{G m(r)}{2 r} \tag{27}
\end{equation*}
$$

From (27) it is seen that when $m(r)$ looks even on the verge of zero near the centre of the galaxy the speed $V$ can become high, too, and this explains so-called missing mass. Again, from (27) we get
$2 r \nu_{T}^{2}=-4 \pi G p r^{3}-G m(r)$, and if $v$ is a constant between $r_{1}$ and $r_{2}$, differentiating this equation and using $v_{T}^{2}\left(r_{1} \leq r \leq r_{2}\right)=-2 \pi G p r^{2}-\frac{G m(r)}{2 r}=\frac{3 M G}{2 r_{e}^{3}} r_{1}^{2}-\frac{G m\left(r_{1}\right)}{2 r_{1}}$ yield

$$
\begin{equation*}
\rho\left(r_{1} \leq r \leq r_{2}\right)=-3 p-\frac{v_{T}^{2}}{2 \pi G r^{2}}=\frac{9 M}{4 \pi r_{e}^{3}}-\frac{3 M}{4 \pi r_{e}^{3} r^{2}} r_{1}^{2}+\frac{m\left(r_{1}\right)}{4 \pi r_{1} r^{2}} \tag{28}
\end{equation*}
$$

which is the condition a typical spiral galaxy with a halo satisfies.
May as well set $r_{1}=n r_{2}(0<n<1)$, then

$$
m\left(r_{2}\right)=m\left(r_{1}\right)+4 \pi \int_{r_{1}}^{r_{2}} \rho r^{2} d r=\frac{3 M}{r_{e}^{3}}\left(1-n^{2}\right) r_{2}^{3}+\frac{m\left(r_{1}\right)}{n}
$$

and in consideration of $0 \leq m\left(r_{2}\right) \leq M$ we concluded that

$$
\begin{equation*}
0 \leq r_{2}^{3} \leq \frac{n M-m\left(r_{1}\right)}{3 n M\left(1-n^{2}\right)} r_{e}^{3} \tag{29}
\end{equation*}
$$

which indicates it is impossible for $r_{2}$ to arrive at the galaxy's edge $r_{e}$ in the case of $n<\sqrt{2 / 3}$ . Obviously, if $\rho$ begins to decrease from $r_{2}$ to $r_{e}$ both $v_{T}$ and $|g|$ begin to increase. Of course, it isn't easy to observe the speed of the particles between $r_{2}$ and $r_{e}$ because near the edge $r_{e}$ matter becomes virtually very thin. The curve in figure 2 describes the situation predicted by (27) and (29), and it is in conformity with recent observational results (Cayrel et al., 2001) .
So far, we conclude that so-called dark matter is just the effect of the negative pressure, and the dark matter (Genzel et al., 2006; Li, 2008, Gaugh, 2008) puzzle has naturally been solved. Of course, so-called dark energy problem is also removed since cosmological constant is reconfirmed as zero and the concept of dark energy becomes unnecessary in the new amendment.


Fig. 2. The velocity distribution diagram

## 7 Motion in centre field and galaxy formation

Equation (14) indicates that not only space is expanding but also celestial bodies or galaxies themselves, that is, like a expanding balloon, the ink prints on it proportionally expand also. To illuminate galaxy formation clearer we investigate the motion in centre field. Let $M$ denote mass of centre body. Generally speaking, its gravitational field is weak, geodesic reduces to Newton's law, for a object moving around the centre body we have

$$
\begin{equation*}
\frac{4 \pi^{2} r^{2}}{T^{2}}=\frac{G M}{r}, \tag{30}
\end{equation*}
$$

where $r$ is the radius of round orbit, $T$ is revolution period. Noticing $M$ to be variable now and to satisfy (18) and using (30) we infer

$$
\begin{gather*}
\Delta r=r(1+\Delta T / T)^{2 / 3} a(t+\Delta t) / a(t)-r \text { from } t \text { to } t+\Delta t . \text { And putting } \Delta t \rightarrow 0 \text { we have } \\
v \equiv \frac{d r}{d t}=r H+\frac{2 r}{3 T} \frac{d T}{d t} . \tag{31}
\end{gather*}
$$

where the final term is explained as perturbation and gravitational radiation. For instance, apply (31) to the motion round today's Earth, for geostationary satellite, neglecting perturbation and gravitational radiation, namely taking $d T_{0}=0$ we find that its orbit radius will increase by $\Delta r_{0}=2.7 \mathrm{~mm}$ in a year. And for the motion of Moon, observations show that its orbit radius increases by 0.38 cm in a year today, then using (31) we conclude that the orbit period $T_{0}$ of Moon will slow by 0.0001 s in a year today. When (31) is used to the edge of a spiral galaxy, it is concluded that the terminuses of spiral arms gradually stretch outward. Of course, other points near the terminuses continuously follow and form involutes. See the following figure 3.


Fig. 3. Sketch map of formation and evolution of spiral arms

Eq. (31) means that separating speed from centre lies on $v=r H$ neglecting perturbation and radiation damp.

It is important to realize that the spin of a system is the composition of orbit motion of many particles, spin and orbit motion do not have essential difference. And for celestial body's expansion, lying on $v=r H$ means its spin period not to change.
Note that the existence of equation (31) doesn't mean the destruction of conservation of angular momentum because mass $M$ is connected with the factor $a(t)$, which embodies the interaction among galaxies, the non-conservation of angular momentum of individual galaxy is admitted.
Again, the fact that space, celestial bodies and galaxies simultaneously expand proportionally links the homogeneity of today's universe in a large range with that of early universe in a small range, because the large range is just the amplification of early the small range. Background radiation has proven early universe to be homogeneous in quite small range. Therefore our conclusion is in accordance with observations.
The following figure 4 is the global picture of galaxy evolution and distribution under $\rho=$ const
in different stages, the earlier, the smaller and the denser. Figure 5 is the picture of galaxies seen by today's telescope, and the farer, the earlier and the evener. Naturally, the microwave background radiation measured today is the compositive effect of various photons emitted by innumerable remote galaxies, whose distances to us are unidentifiable, which comprised infinitely deep thin gas and can absorb any frequency photon and therefore possess black body character.


Fig. 4. The global picture of galaxy evolution and distribution in different stages


Fig. 5. The actual picture of galaxies seen by today's telescope

Note that the horizon at moment $t>0$ is now from (15)

$$
d_{h}(t) \equiv a(t) \int_{0}^{t} \frac{1}{a(t)} d t=\sin \left(t \sqrt{4 \pi G \rho_{0} / 3}\right) \int_{0}^{t} d t / \sin \left(t \sqrt{4 \pi G \rho_{0} / 3}\right)=\infty
$$

So-called horizon puzzle or homogeneity puzzle does not exist in the present theory framework at all. Naturally, the microwave background radiation measured today is the compositive effect of various photons emitted by unnmberable galaxies remote, whose distances to us are unidentifiable, which comprised infinitely deep thin gas and could absorb any frequency photon and therefore possess black body feature.

Note that the state that horizon vanishes is unobservable though $d_{h}(t)=0$ for $t=0$, because any observation needs a lag of time $\Delta t$

## 8 Quantum process of continuous creation of matter in celestial bodies

$P=-\rho$ tells us that the negative pressure in celestial bodies is actually a negative energy field, and $p$ and $\rho$ excite with each other and generate simultaneously. Connecting with particle physics it is naturally deduced that in celestial bodies many particle-antiparticle pairs (including neutron and antineutron, proton and antiproton, electron and positron and so on) can ceaselessly produce and annihilate, the antiparticles lie in negative energy level (can't be observed), the particles lie in positive energy level, and the absolute value of energy of particle and antiparticle is equal. Let $\Delta t$ denote the lifetime of a kind of particle-antiparticle pairs, namely the average time from production to annihilation, according to uncertain principle the range $\Delta E$ of energy satisfies

$$
\begin{equation*}
\Delta E \geq \frac{\hbar}{2 \Delta t} \tag{32}
\end{equation*}
$$

which shows that instantaneous energy of new particle may be very high. Note that not all of the particles annihilate as soon as they come into being, only those which don't not have opportunity in the time $\Delta t$ to react with the surrounding particles or to collide and change their energy can
annihilate, once the reaction with other particles or the collision occur the annihilation no longer carry out, and in this case the negative energy field detains a negative energy antiparticle while the particle becomes constituents of matter. Therefore, the negative energy field is too a quantum field to consist of various negative energy antiparticles. Of course, an antiparticle of negative energy $-\mathcal{E}$ can be excited to positive energy $\mathcal{E}$ by a meson of energy $2 \mathcal{E}$ and becomes the antiparticle that can be observed. For no other reason than that many antiparticles lie in negative energy level and can't be observed, we perceive that particles and antiparticles aren't symmetrical. As a result of general relativity, equation (14) in section 4 exposes already that matter and antimatter are symmetrical.
Obviously the negative pressure field provides energy source of star radiation, not only thermal nuclear reactions, therefore the mystery of solar neutrino doesn't exist in the new theory framework.
And considering of tunneling effect in quantum theory, many nuclear reactions are able to complete slowly in celestial bodies even if the temperature ( average kinetic energy of particles ) is low, which implies that in the case of low temperature elements can also compose. As for what kind of nuclear reaction is in evidence, this depends on temperature of celestial bodies. And as a result, the abundance of elements in a celestial is the effect of various nuclear reaction for long time.

For a celestial body of temperature T, we may as well treat all atoms in it as an open thermodynamic system, whose giant distribution function according to quantum statistics is

$$
\rho=\exp \left(-\Psi-\sum_{i=1}^{k} \alpha_{i} N_{i}-\beta E\right)
$$

Where $N_{i}$ denotes the number of atoms of i-th kind element. And let $m_{i}$ denote its mass, the total energy $E=\sum_{i=1}^{k} N_{i} m_{i}$, then the average value of atom number of element of $j$-th kind reads

$$
\begin{aligned}
\bar{N}_{j} & =\frac{\sum_{N_{1}=0}^{\infty} \sum_{N_{2}}^{\infty} \cdots \sum_{N_{k}}^{\infty} \exp N_{j}\left[-\psi-\sum_{i=1}^{k} N_{i}\left(\alpha_{i}+\beta m_{i}\right)\right]}{\sum_{N_{1}=0}^{\infty} \sum_{N_{2}}^{\infty} \cdots \sum_{N_{k}}^{\infty} \exp \left[-\psi-\sum_{i=1}^{k} N_{i}\left(\alpha_{i}+\beta m_{i}\right)\right]}=-\frac{1}{m_{j}} \frac{\partial}{\partial \beta} \operatorname{In} \sum_{N_{j}=0}^{\infty} \exp \left(-\alpha_{j}-\beta m_{j}\right) N_{j} \\
& =\frac{1}{m_{j}} \frac{\partial}{\partial \beta} \operatorname{In}\left(1-e^{-\alpha_{j}-\beta m_{j}}\right)=\frac{1}{\exp \left(\alpha_{j}+\beta m_{j}\right)-1}=\frac{1}{\exp \frac{m_{j}-\mu_{j}}{k T}-1}
\end{aligned}
$$

Here $\mu_{i}$ amounts to the chemical potential of the group, $T$ is the temperature of the celestial body, namely average kinetic energy of all atoms, $k$ is Boltzmann constant. From above relation we have for arbitrary two elements A and B

$$
\begin{equation*}
\frac{\bar{N}_{A}}{\bar{N}_{B}}=\frac{\exp \frac{m_{B} c^{2}-\mu_{B}}{k T}-1}{\exp \frac{m_{A} c^{2}-\mu_{A}}{k T}-1} \tag{33}
\end{equation*}
$$

(33) decides the abundance of elements in a celestial body. Observations of astronomy show that element abundance is different for different celestial, which is consistent with (33). Observations of astronomy show that the abundance of elements is in accordance seen from large scope, which implies both temperature and chemical potential are uniform seen from large scope. Observations of astronomy show that all elements in other body can be found out on the earth, which implies that the origin of various elements is the same, namely originate production and annihilatation of particle-antiparticle pairs.

## 9 Conclusions

Density and pressure of universe do not change all along (Massimiliano et al., 2001), big bang didn't exist and matter in universe is produced continuously and slowly. With cosmic expansion celestial bodies and galaxies expand too, which is just the fundamental mechanism of celestial body and galaxy formation. The dark matter to appear as negative pressure is just the antimatter that lies in negative level, it cannot exist alone and must hide in the usual matter.

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## Appendices

## A: the deduction of (20) and (24)

According to description of general relativity, in the case of static spherical symmetry, in standard coordinate system the form of invariant line element is written as

$$
d s^{2} \equiv-d \tau^{2}=-B(l) d t^{2}+A(l) d l^{2}+l^{2}\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right)
$$

where $l$ is called standard radial coordinate, space-time coordinate

$$
x^{\mu}=\left(x^{0}, x^{1}, x^{2}, x^{3}\right)=(t, l, \theta, \varphi) . \text { And } g_{00}=g_{t t}=-B(l), g_{11}=g_{r r}=A(l)
$$

$g_{22}=g_{\theta \theta}=l^{2}, \quad g_{33}=g_{\varphi \varphi}=l^{2} \sin ^{2} \theta$, the other components are equal to zero. From the definition of inverse Matrix we work out $g^{00}=-\frac{1}{B}, g^{11}=\frac{1}{A}, g^{22}=\frac{1}{l^{2}}, g^{33}=\frac{1}{l^{2} \sin ^{2} \theta}$, the others are equal to zero. And form $\Gamma_{\mu \nu}^{\rho}=\frac{1}{2} g^{\rho \sigma}\left(\frac{\partial g_{\sigma \mu}}{\partial x^{\nu}}+\frac{\partial g_{\sigma \nu}}{\partial x^{\mu}}-\frac{\partial g_{\mu \nu}}{\partial x^{\sigma}}\right)$, we work out
$\Gamma_{11}^{1}=\frac{A^{\prime}}{2 A} \quad \Gamma_{01}^{0}=\frac{B^{\prime}}{2 B}, \quad \Gamma_{33}^{2}=-\sin \theta \cos \theta, \quad \Gamma_{23}^{3}=\cot \theta, \quad \Gamma_{33}^{1}=-\frac{l}{A} \sin ^{2} \theta$, $\Gamma_{12}^{2}=\Gamma_{13}^{3}=\frac{1}{l}, \quad \Gamma_{22}^{1}=-\frac{1}{2} g^{11} \frac{\partial g_{33}}{\partial l}=-\frac{l}{A} \sin ^{2} \theta, \quad \Gamma_{00}^{1}=\frac{B^{\prime}}{2 A}$, the others are zero, where $A^{\prime}=\frac{d A}{d l} B^{\prime}=\frac{d B}{d l}$. And form $R_{\mu \nu}=\Gamma_{\mu \sigma}^{\sigma},_{\nu}-\Gamma_{\mu \nu, \sigma}^{\sigma}+\Gamma_{\mu \sigma}^{\tau} \Gamma_{\tau \nu}^{\sigma}-\Gamma_{\mu \nu}^{\tau} \Gamma_{\sigma \tau}^{\sigma}$, we work out $R_{00}=-\frac{B^{\prime \prime}}{2 A}+\frac{B^{\prime}}{4 A}\left(\frac{A^{\prime}}{A}+\frac{B^{\prime}}{B}\right)-\frac{B^{\prime}}{l A^{\prime}}$,
$R_{22}=\frac{l}{2 A}\left(-\frac{A^{\prime}}{A}+\frac{B^{\prime}}{B}\right)+\frac{1}{A}-1, \quad R_{33}=\sin ^{2} \theta R_{22}, \quad R_{11}=\frac{B^{\prime \prime}}{2 B}-\frac{B^{\prime}}{4 B}\left(\frac{A^{\prime}}{A}+\frac{B^{\prime}}{B}\right)-\frac{A^{\prime}}{l A}$, the others are zero. On the other hand $T_{\mu \nu}=(\rho+p) U_{\mu} U_{v}+p g_{\mu \nu}, \quad g^{\mu \nu} U_{\mu} U_{\nu}=-1$,

$$
T=g^{\mu \nu} T_{\mu \nu}=3 p-\rho,
$$

and for the case of static spherical symmetry $p=p(l), \rho=\rho(l), U_{0}=-\sqrt{B}, U_{i}=0$, then we work $\quad$ out $T_{00}-\frac{T}{2} g_{00}=\frac{B(3 p+\rho)}{2}, T_{33}-\frac{T}{2} g_{33}=l^{2} \sin ^{2} \theta \frac{(\rho-p)}{2}$, $T_{22}-\frac{T}{2} g_{22}=\frac{l^{2}(\rho-p)}{2}, T_{11}-\frac{T}{2} g_{11}=\frac{A(\rho-p)}{2}$ the other corresponding components are zero. Field equation (11) is equivalent to $R_{\mu \nu}=4 \pi G\left(T_{\mu \nu}-\frac{1}{2} T g_{\mu \nu}\right)$, we get the following three independent equations:

$$
\left\{\begin{array}{c}
R_{00}=-\frac{B^{\prime \prime}}{2 A}+\frac{B^{\prime}}{4 A}\left(\frac{A^{\prime}}{A}+\frac{B^{\prime}}{B}\right)-\frac{B^{\prime}}{l A^{\prime}}=2 \pi G(\rho+3 p) B \\
R_{11}=\frac{B^{\prime \prime}}{2 B}-\frac{B^{\prime}}{4 B}\left(\frac{A^{\prime}}{A}+\frac{B^{\prime}}{B}\right)-\frac{A^{\prime}}{l A}=2 \pi G(\rho-p) A \\
R_{22}=\frac{l}{2 A}\left(-\frac{A^{\prime}}{A}+\frac{B^{\prime}}{B}\right)+\frac{1}{A}-1=2 \pi G(\rho-p) l^{2}
\end{array}\right.
$$

And the other corresponding equations are identities. Then we have

$$
\frac{R_{00}}{2 B}+\frac{R_{11}}{2 A}+\frac{R_{22}}{l^{2}}=-\frac{1}{l^{2}}+\frac{1}{A l^{2}}-\frac{A^{\prime}}{l A^{2}}=4 \pi G \rho, \quad \text { namely }\left(\frac{l}{A}\right)^{\prime}=1+4 \pi G \rho l^{2},
$$

and since $A(0)$ is limited, we infer $A(l)=\left(1+\frac{G \omega(l)}{l}\right)^{-1}$, where $\omega(l) \equiv 4 \pi \int_{0}^{l} \rho(l) l^{2} d l$.
On the other hand, the conservation law $T_{\mu ; \nu}^{v}=0$ gives $\frac{B^{\prime}}{B}=-\frac{2 p^{\prime}}{\rho+p}$, then from
$R_{22}=\frac{l}{2}\left(1+\frac{G \omega(l)}{l}\right)\left[\frac{G}{l^{2}}\left(l \omega^{\prime}-\omega\right)(1+G \omega)^{-1}-\frac{2 p^{\prime}}{\rho+p}\right]+\left(1+\frac{G \omega}{l}\right)-1=2 \pi G(\rho-p) l^{2}, \quad$ after being simplified

$$
\frac{d p}{d l}=G(p+\rho)\left(2 \pi l^{3} p+\frac{\omega}{2}\right)\left(l^{2}+l G \omega(l)\right)^{-1} .
$$

And again, from $\frac{B^{\prime}}{B}=-\frac{2 p^{\prime}}{\rho+p}=-2 G\left(2 \pi l^{3} p+\frac{\omega}{2}\right)\left(l^{2}+l G \omega(l)\right)^{-1}$, we obtain

$$
B(l)=\exp \left[\mathrm{C}_{2}+\int_{l}^{l_{e}} f(l)\left(1+\frac{\omega(l)}{l}\right)^{-1} d l\right],
$$

Where $f(l) \equiv \frac{G}{l^{2}}\left[4 \pi l^{3} p(l)+\omega(l)\right]$, and constant $C_{2}=\operatorname{In}\left(1-\frac{2 G M}{l_{e}}\right)$, it makes sure $B(l)$ is continuous on the bound $r_{e}$ (surface of source). Note that the value of $l\left(r_{e}\right)$ on the bound is determined by (23).

B: the deduction of luminosity distance $d_{L}=(1+Z) \int_{0}^{l_{a}} \frac{d l}{\sqrt{1-k l^{2}}}$,
At the moment $t$ proper distance of a galaxy is defined as $d_{p}=a(t) \int_{0}^{l_{a}} \frac{d l}{\sqrt{1-k l^{2}}}$. Let a telescope of area $A$ faces the galaxy. Within time $\delta t_{e}$ the galaxy emitted $n$ photons of total energy $n h \nu_{e}$, and within time $\delta t_{0}$ they arrive at the telescope. Spectrum radiate power of galaxy is defined as $L \equiv \frac{n h \nu_{e}}{\delta t_{e}}$. Power received by telescope is $p=\frac{n h v_{0}}{\delta t_{0}} \frac{A}{4 \pi d_{p}^{2}\left(t_{0}\right)}$. Using $v_{0}=\frac{v_{e} a\left(t_{e}\right)}{a\left(t_{0}\right)}$ and $\frac{1}{\delta t_{0}}=\frac{a\left(t_{e}\right)}{\delta t_{e} a\left(t_{0}\right)}$, we have

$$
p=\frac{n h v_{0}}{\delta t_{0}} \frac{A}{4 \pi d_{p}^{2}\left(t_{0}\right)}=\frac{n h v_{e} a^{2}\left(t_{e}\right)}{\delta t_{e} a^{2}\left(t_{0}\right)} \frac{A}{4 \pi d_{p}^{2}\left(t_{0}\right)}=\frac{L A a^{2}\left(t_{e}\right)}{4 \pi a^{2}\left(t_{0}\right) d_{p}^{2}\left(t_{0}\right)} .
$$

Vision luminosity received by telescope is defined as $l \equiv \frac{p}{A}=\frac{L a^{2}\left(t_{e}\right)}{4 \pi a^{2}\left(t_{0}\right) d_{p}^{2}\left(t_{0}\right)}$. We know that vision luminosity of light source in Euclidean space is $l=\frac{L}{4 \pi d^{2}}$, and generalizing
the definition to curve space, luminosity distance $d_{L} \equiv \sqrt{\frac{L}{4 \pi l}}=\frac{a\left(t_{0}\right)}{a\left(t_{e}\right)} d_{p}\left(t_{0}\right)$. Using $\frac{a\left(t_{0}\right)}{a\left(t_{e}\right)}=1+Z \quad$ and $\quad$ putting $\quad a\left(t_{0}\right)=1, \quad$ finally $\quad$ we $\quad$ obtain $d_{L}=(1+Z) d_{p}\left(t_{0}\right)=(1+Z) \int_{0}^{l_{a}} \frac{d l}{\sqrt{1-k l^{2}}}$
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