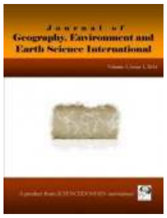

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The Use of Exponential Distribution Model to Estimate Recurrence Periods of Earthquakes in Zimbabwe

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Authors' contributions

This work was carried out in collaboration between all authors. Author AAA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors JUA and HAA managed the analyses of the study. Author JAU managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

This study estimated recurrence periods of earthquakes in Zimbabwe using exponential distribution model. The data for this study were extracted from a catalogue, the Advanced National Seismic System (ANSS) owned by Northern California Earthquake Data Centre UC Berkeley, USA. The selected data consisted of earthquake events with magnitude $M_b \geq 4.0$ for the study area from 1st January, 1901 to 31st December, 2001 (100 years) with focal depth from 0 – 700 km within latitudes 15°S – 22°S and longitudes 25°E – 34°E. A total number of 81 events were employed in the study. The formulated hypothesis was tested using Chi square test and Independent t-test. The findings of this study revealed that experimental distribution of earthquakes have no significant difference with theoretical distribution of earthquakes in Zimbabwe. This implies that Zimbabwe earthquake data follow exponential distribution. The return periods for magnitude 4.2 and 6.2 were estimated to be

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4.00 and 47.48 years respectively. It has been observed that as the magnitude increases towards higher magnitude, the return period increases except at magnitude 4.7 where it decreased. Therefore, the occurrence of minor to light earthquakes is more frequent than stronger ones: therefore, the probability of occurrence of earthquakes of low magnitude (up to 4.0) is higher than for earthquakes with magnitude of 6.0 and above. As a result, Zimbabwe may not likely experience any serious earthquake (magnitude 6.0 or greater) until the year 2048, considering that the last 6.0 magnitude event - with an estimated return period of 47.48 years - occurred in 2001. Nevertheless, earthquake occurrence cannot be predicted with certainty yet: earthquakes are in fact naturally unpredictable, due to sensitivity of catalogues to small events, saturation of magnitudes and differences in data collection by seismic stations and networks.

Keywords: Earthquake; recurrence period; hypothesis; Zimbabwe; probability.

1. INTRODUCTION

Earthquake is one of the most dangerous catastrophes facing man throughout the human history. Man has made several efforts to unravel the processes, causes and ways of predicting earthquake, but global records of earthquake damages are worrisome. Earthquakes have caused deaths, destroyed properties worth billions of dollars and rendered people homeless in different parts of the world. Among the best-known earthquake events are: the 1994 Northridge earthquake in California; the 1995 great Haushin Kobe earthquake in Japan [1]; the 1999 Izmit earthquake in Turkey; the 1993 Killari earthquake in India; and the 1999 Chi-chi earthquake in Taiwan.

Several studies have been conducted on the probability distribution of earthquakes globally. [2] employed Weibull, gamma, lognormal, and exponential to study earthquake return intervals in Japan and its environ [3,4]. [5,6,7,8,9,10] employed similar procedures for their studies. The Gaussian distribution [11], the negative binomial distribution [12,13,14], the Pareto group of distributions [15,16], the generalized gamma distribution [17], the Brownian passage time distribution [18,14], the Rayleigh distribution [16,7], the inverse Weibull distribution [9], and the exponentiated exponential distribution [19,10] has also been successfully applied by researchers.

Zimbabwe has moderate earthquake distribution pattern with predominant earthquakes happening near Zimbabwe – Mozambique border, Nyamandlovu area and the northern region. Some of these events happened in areas that were initially considered as aseismic. Due to the increase in industrial development, urbanization of the population, number of high rise buildings, construction of dams and mining activities, therefore there is a need to investigate the seismicity of Zimbabwe.

The objective of this study is to determine earthquake return periods using exponential distribution model and to test the significant difference between the theoretical and experimental distribution of earthquakes in Zimbabwe.

1.1 Seismicity of the Study Area

Zimbabwe is situated within the southern end of East African Rift system [20,21,22,23]. Seismicity of Zimbabwe consists of three seismic zones. These include: the Eastern region, the Zambezi basin region and the central region. Earthquake activities are commonly concentrated east in the Mozambique border, northwest in the Deka fault zone and Lake Kariba region in Mid-Zambezi basin.

Earthquakes of magnitude greater than 5.0 happened in the Mid–Zambezi basin [24,25]. Research on incipient rifting in the Mid-Zambezi basin region proved that tectonic activity in this region corresponds with the one in north side of East African rift system [21,22]. The South-eastern border region of Zimbabwe constitutes the Western flank of the rift extension from Lake Malawi [26].

The Seismicity of the Zambezi region before the construction of Lake Kariba is not known. The monitoring equipment was absent prior to the construction of the reservoir. Only few notable events were observed in December, 1958. The occurrence earthquake events 50 km to the north of Lake Kariba dam supported the existence of natural tectonic activity.

1.2 Research Hypothesis

The hypothesis was formulated to serve as a guide to the study.

H_o : Experimental distribution of earthquakes has no significant difference with theoretical distribution of earthquake in Zimbabwe.
H_1 : Experimental distribution of earthquakes has significant difference with theoretical distribution of earthquake in Zimbabwe.

2. MATERIALS AND METHODS

2.1 Source of Data

The data used were extracted from the Advanced National Seismic System (ANSS), a website of Northern California Earthquake Data Centre UC Berkeley, USA. The selected data consisted of earthquakes with magnitude $M_b \geq 4.0$ for study area from 1st January, 1901 to 31st December, 2001 (100 years) with focal depth from 0 – 700 km within latitudes 15°S – 22°S and longitudes 25°E – 34°E (Fig. 1). The data set is made up of date of occurrence, origin time, coordinates of epicenter, focal depth magnitude and event ID.

2.2 Exponential Distribution Model

In estimating the recurrence of earthquake events, magnitude plays an important role. Magnitude is continuous random variable that has lower limit θ, and does not have clear defined theoretical limit.

The expression of probability density function of x random variable in terms of exponential function is given by [28,29,30]:

$$F_m(x) = \lambda e^{-\lambda(x-\theta)} \quad \lambda > 0 \quad \theta \leq x < +\infty \tag{1}$$

Where x is the expected magnitude of earthquake M.

$$\lambda = (\bar{x} - \theta)^{-1} \tag{2}$$

$$\lambda = \frac{1}{(\bar{x} - \theta)} \tag{3}$$

Where $\bar{x}$ the mean magnitude of earthquake and θ is the lower limit.

The distribution function of x random variable is given by:

$$F_m(x) = \int_0^x \lambda e^{-\lambda(u-\theta)} du \tag{4}$$

Integrating (4), yields

$$F_m(x) = 1 - e^{-\lambda(u-\theta)} \quad \lambda>0, \ \theta \leq x < +\infty \tag{5}$$

2.3 Chi-Square Test

Chi-square χ^2 test was used to test the formulated hypothesis. This test enables us to see how well does the theoretical distribution fit to the observed (experimental) data. When calculated χ_h^2 value is less than critical χ_i^2, the hypothesis is said to be true. Otherwise, the hypothesis is rejected [29].

The following equation was used:

$$\chi^2 = \sum_{i=k}^{k} \frac{[F_M(o_i) - F_{Mb}(o_i)]^2}{F_{Mb}(o_i)} \tag{6}$$

Where k is the number of class, $F_M(O_i)$ is the observed probabilities and $F_{Mb}(O_i)$ is the expected probabilities.

2.4 Independent T-test

The hypothesis was also tested using independent t-test. It was employed because it helps us to judge the significance difference between means of two groups or samples. The t-test is given by:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{SD_1^2}{n_1} + \frac{SD_2^2}{n_2}}} \tag{7}$$

Where $\bar{X}_1$ is the mean of group one
$\bar{X}_2$ is the mean of group two
SD_1 is the standard deviation of group one
SD_2 is the standard deviation of group two
n_1 is the number of items in group one
n_2 is the number of items in group two

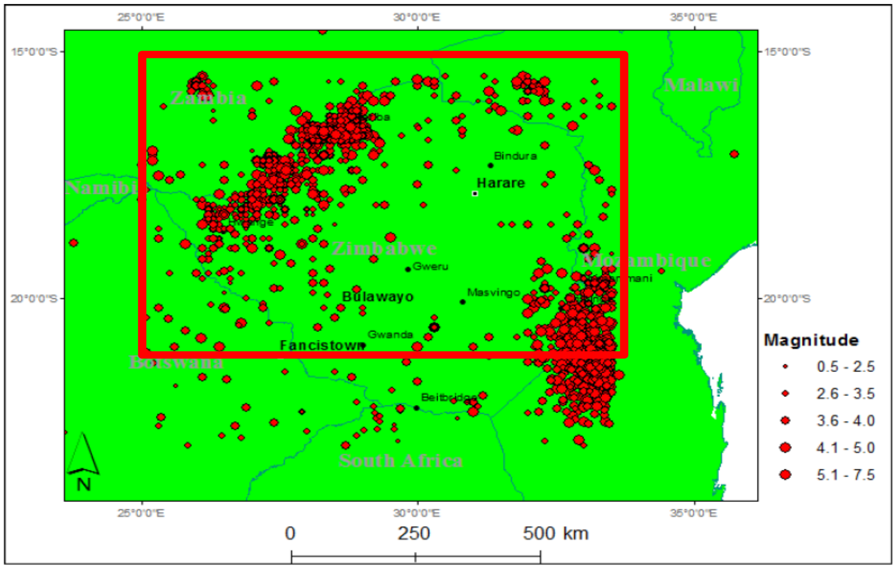

Fig. 1. Map of Zimbabwe with rectangle showing the study area (Modified from [27])

3. RESULTS AND DISCUSSION

3.1 Test of Hypothesis

Experimental distribution of earthquakes has no significant difference with theoretical distribution of earthquakes in Zimbabwe.

This hypothesis was tested using chi-square test with unconstrained degrees of freedom DF=3 at α= 0.05 level of significance [29].

Degrees of Freedom (DF) = 3

Critical χ^2 *= 7. 8147*

The result in Table 1 shows that the critical Chi-square value of 7.8147 is greater than the calculated chi-square χ_h^2 value of 0.001279. With this result, the null hypothesis is accepted and the alternative is rejected. This implies that experimental distribution of earthquakes has no significant difference with theoretical distribution of earthquakes in Zimbabwe.

Table 3 shows that the probability or p-value or significant value of 0.958 between experimental and theoretical distribution of earthquakes is not significant. Therefore, the null hypothesis is accepted and the alternative hypothesis is rejected. Hence, experimental distribution of earthquakes has no significant difference with theoretical distribution of earthquakes in Zimbabwe.

Table 1. Chi-square test table

$F_{mg}(x)$	$F_{mb}(x)$	$F_{mg}(x) - F_{mb}(x)$	$(F_{mg}(x) - F_{mb}(x))^2$	$\frac{[F_{mg}(x) - F_{mb}(x)]^2}{F_{mb}(x)}$
0.309	0.309	0.000	0.000	0.000000
0.729	0.726	0.003	9E-06	1.23E-05
0.889	0.891	-0.002	4E-06	4.5E-06
0.988	0.957	0.031	0.000961	0.000973
1.000	0.983	0.017	0.000289	0.000289
				0.001279

Table 2. Descriptive statistics for experimental and theoretical distribution of earthquakes in Zimbabwe

	Values	N	Mean	Std. deviation	Std. error mean
$F_{mg}(x)$	1.00	5	.7830	.28633	.12805
$F_{mb}(x)$	2.00	5	.7732	.27811	.12438

Table 3. Independent t-test of the relationship between experimental and theoretical distribution of earthquakes in Zimbabwe

		Levene's test for equality of variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. Error difference	95% confidence interval of the difference	
									Lower	Upper
$F_{mg}(x)$	Equal variances assumed	.004	.949	.055	8	.958	.00980	.17851	-.40185	.42145
$F_{mb}(x)$	Equal variances not assumed			.055	7.993	.958	.00980	.17851	-.40191	.42151

Table 4. Distribution of earthquakes based on frequency

Class	Lower class	Upper class	Class midpoint	Frequency(F)	Percentage (%)
4.0 - 4.4	4.0	4.4	4.2	25	0.309
4.5 - 4.9	4.5	4.9	4.7	34	0.420
5.0 - 5.4	5.0	5.4	5.2	13	0.160
5.5 - 5.9	5.5	5.9	5.7	8	0.099
6.0 - 6.4	6.0	6.4	6.2	1	0.012

Table 5. Earthquake magnitude class midpoint and frequency

Class midpoint (x)	Frequency (F)	Fx
4.2	25	105
4.7	34	159.8
5.2	13	67.6
5.7	8	45.6
6.2	1	6.2
	81	**384.2**

$$\text{The Arithmetic mean } \bar{x}=\frac{\Sigma fx}{fx}=\frac{384.2}{81}=4.74$$

$$\bar{x}=4.74$$

Table 6 shows the frequency of earthquake, class midpoint, the percentage, the empirical values, the theoretical values and difference of values.

To get the empirical values, the first percentage value is kept constant and the other percentage values are added sequentially to each of the values to get the corresponding results.

While to get the theoretical values, equation (5) was employed.

$$\text{But } \lambda=\frac{1}{(\bar{x}-\theta)}$$

$$\bar{x}=4.74$$

$$\lambda=\frac{1}{(4.74-4.2)}=\frac{1}{0.54}=1.85$$

$$\lambda=1.85$$

θ is the lower bound of first class and in this case it is 4.0.

Substituting the numerical value of λ=1.85 and θ = 4.0 into eqn(5) yields:

$$F_m(x)=1-e^{-1.85\,(x-4.0)} \tag{8}$$

From equation (8), the theoretical values can be obtained.

From Table 7, probability of earthquakes with different magnitudes $F_m(x)$ can be obtained using the cumulative of the theoretical values $F_{mb}(x)$. To get the annual expected recurrence frequency, we multiply F_1 by $F_m(x)$, where F_1 is the ratio of the sum of earthquake frequency (81) to the examined time period (100)

$$F_1=\frac{81}{100}=0.81$$

To get the average recurrence period, we use the reciprocal of the annual expected earthquake frequency which gives $\frac{1}{AEEF}$

AEEF = Annual Expected Earthquake Frequency.

The hypothesis of this study revealed that experimental distribution of earthquake has no significant difference with theoretical distribution of earthquakes in Zimbabwe. The recurrence periods of earthquakes for some magnitudes were estimated (Table 7). It is observed that recurrence periods for magnitude 4.2 and 6.2 were estimated to be 4.00 and 47.48 years respectively. As the magnitude increases

Table 6. Distribution values based on the constructed model for experimental and theoretical probabilities

Midpoint (x)	N	Percentage (%)	Empirical $F_{mg}(x)$	Theoretical $F_{mb}(x)$	Difference of values
4.2	25	0.309	0.309	0.309	0.000
4.7	34	0.420	0.729	0.726	0.003
5.2	13	0.160	0.889	0.891	-0.002
5.7	8	0.099	0.988	0.957	0.031
6.2	1	0.012	1.000	0.983	0.017

Table 7. Return periods of different class intervals

Class midpoint value	Theoretical value F_{mb} (x)	F_m(x)	Annual expected recurrence frequency	average recurrence period
4.2	0.309	0.309	0.25029	4.00
4.7	0.726	0.417	0.33777	2.96
5.2	0.891	0.165	0.13365	7.48
5.7	0.957	0.066	0.05346	18.71
6.2	0.983	0.026	0.02106	47.48

towards higher magnitude, the return period increases except at magnitude 4.7 where it decreased. Therefore, the occurrence of minor to light earthquakes is more frequent than the strong earthquakes. This means that the probability of occurrence is higher for the earthquakes with magnitude 4.0 and below than it is for the earthquakes with magnitude of 6.0 and above. Although it is low, it cannot be predicted with certainty since earthquakes are naturally unpredictable.

4. CONCLUSION

The result of the hypothesis in this study revealed that experimental distribution of earthquakes has no significant difference with theoretical distribution of earthquakes in Zimbabwe. This indicates that magnitude random variable of Zimbabwe earthquake data follow the exponential distribution. The recurrence periods for earthquakes of magnitude 4.2 and 6.2 were estimated to be 4.00 and 47.48 years respectively. This indicates that the probability of occurrence is higher for the earthquakes with magnitude 4.0 and below than it is for the earthquakes with magnitude of 6.0 and above. As a result, Zimbabwe may not likely experience any serious earthquake of magnitude 6.0 and above till the year 2048 since 6.0 magnitude last occurred in 2001 with the probable return period of 47.48 years. Nevertheless, it cannot be predicted with certainty since earthquakes are naturally unpredictable due to sensitivity of earthquake catalogues to small events, saturation of earthquake magnitudes and variation in seismic data collection by different seismic stations and networks. The implication of these findings shows that it furnishes seismologists with information to know areas/regions with short and long recurrence periods. This will help in planning and preparation for the future occurrence of an earthquake event.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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